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جامعة السودان للعلوم والتكنولوجيا

كلية الهندسة – قسم الهندسة الالكترونية

المادة: الحقول الكهرومغناطيسية | اختبار رقم (1)

الفصل الدراسي السادس	ثالثة الكترونيات	أستاذ المادة: د.محمد حسين	الزمن: ساعة

Question 1:

A uniform volume charge density of $\rho_v = -5 \times 10^{-6} e^{-10^5 \rho z}\ C/m^3$, what is total charge is enclosed in the volume $0 \le \rho \le 1\ cm$, $0 \le \emptyset \le 2\pi$, $2 \le z \le 4\ cm$.

Question 2:

Let a point charge $Q_1 = 25\ nC$ be located at $p_1(4,-2,7)$ and a charge $Q_2 = 60\ nC$ be at $p_2(-3,4,-2)$. At which point on the y axis is $E_x = 0$.

−6.89

−22.11

Question 3:

Given the electric flux density, $D = (2x+1)y^2{}_{a_x} + 2x(x+1)_{a_y}\ C/m^2$ evaluate the total charge enclosed in the surface:

1. $x = 5$; $-2 \le y \le 2$, $-2 \le z \le 2$.
2. $y = 2$, $-5 \le x \le 5$; $-2 \le z \le 2$.

$\psi = \int D$

"""مع التمنيات بالتوفيق"""

620

Question 1:

$$Q = \int_{0.02}^{0.04}\int_{0}^{2\pi}\int_{0}^{0.01} -5\times10^{-6} e^{-10^5 \rho z} \rho \, d\rho \, d\phi \, dz$$

We integrate first with respect to ϕ since it is so easy

$$Q = \int_{0.02}^{0.04}\int_{0}^{0.01} -10^{-5}\pi e^{-10^5 \rho z} \rho \, d\rho \, dz$$

and then with respect to z because this will simplify the Last integration with respect to ρ

$$Q = \int_{0}^{0.01} \left(\frac{10^{-5}\pi}{-10^5 \rho} e^{-10^5 \rho z} \rho \, d\rho \right)_{z=0.02}^{z=0.04}$$

$$= \int_{0}^{0.01} -10^{-5}\pi \left(e^{-2000\rho} - e^{-4000\rho} \right) d\rho$$

Finally

$$Q = -10^{-10}\pi \left(\frac{e^{-2000\rho}}{-2000} - \frac{e^{-4000\rho}}{-4000} \right)_{0}^{0.01}$$

$$= -10^{-10}\pi \left(\frac{1}{2000} - \frac{1}{4000} \right) = \frac{-\pi}{40} = 0.0785 \text{ pC}$$

Question 2 :

P_3 is now at $(0, y, 0)$, so: $R_{13} = -4\,a_x - (y+2)a_y - 7a_z$

and $R_{23} = 3a_x + (y-4)a_y + 2a_z$. Also, $|R_{13}| = \sqrt{65 + (y+2)^2}$

and $|R_{23}| = \sqrt{13 + (y-4)^2}$.

Now the x component of E at the new P_3 will be:

$$E_x = \frac{10^{-9}}{4\pi\epsilon_0}\left[\frac{25 \times (-4)\ a_x}{[65 + (y+2)^2]^{1.5}} + \frac{60 \times 3\ a_x}{[13 + (y-4)^2]^{1.5}}\right]$$

To obtain $E_x = 0$, we require the expression in the large brackets to be zero. This expression simplifies the following quadratic:

$$0.48\,y^2 + 13.72\,y + 73.10 = 0$$

which yields the two values: $y = -6.89,\ -22.11$

1. $Q = \oint D_s \cdot ds$

$$\Rightarrow \int_{-2}^{2}\int_{-2}^{2} \left(\left((2x+1)y^2 a_x + 2x(x+1) a_y\right) \cdot dy\,dz\, a_x\right)$$

$$\Rightarrow \int_{-2}^{2}\int_{-2}^{2} (2x+1)\, y^2\, dy\, dz$$

$$= (2x+1) \int_{-2}^{2} y^2\, dy \int_{-2}^{2} dz$$

$$= \left((2x+1) * \frac{16}{3} * 4\right)_{x=5} = 234.66 \times 10^{-3}\, C$$

$$= \underline{2.35\, C}$$

2. $Q = \oint D_s . ds$

$$\Rightarrow \int_{-5}^{5}\int_{-2}^{2} \left(\left((2x+1)y^2 a_x + 2x(x+1) a_y\right) \cdot dx\,dz\, a_y\right)$$

$$\Rightarrow \int_{-5}^{5}\int_{-2}^{2} 2x(x+1)\, dx\, dz$$

*

$$= \int_{-5}^{5} (6x^2 + 2x)\, dx \int_{-2}^{2} dz$$

$$= \left(\left(\left(\frac{280}{3} + \frac{280}{3}\right) + 20\right) * 4\right)_{y=2} = 806.66 \times 10^{-3} = 807$$

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Sudan University of science and technology

College of engineering - Electronics department

Date:- 13/7/2009

Year: 3rd electronics — Allowed Time 1:30h

Subject: Electromagnetic field — Test 1

!!Answer All Questions!!

Q1:-

Two vectors fields are:-

$F = -10_{ax} + 20(y-1)_{ay}$ and $G = 2x^2y_{ax} - 4_{ay} + z_{az}$ for the point P(2,3,-4) find?

(a) $|F|$, (b) $|G|$.

(c). Unit vector in direction of F-G.

(d). Unit vector in direction of F+G.

Sap. + 1109

Q2:-

(a) determine the Cartesian component of the vector from A(5, 110°, 200°) to B(7, 30°, 70°). 7

(b) Find the spherical components of the vector at P (2,-3, 4) extending to Q (-3, 2, 5).

(c) If $D = 5_{ar} + 4_{a\phi}$ find D.ap at M (x=1, y=2, z=3).

Q3:-

Four 10-nC positive charges are located in the Z=0 plane at the corners of a square 8-cm on aside.

A fifth 10-nC positive charge is located at point 8-cm distance from each of other charges, calculate?

(a) The magnitude of electric field at the point of fifth charge.

(b) The total force cross fifth charge.

(Note: all charges located in free space).

Q4:-

Volume charge density is located in free space as $\rho_v = 2e^{(-1000r)}$ nC/m³ for $0 < r < 1$ mm and $\rho_v = 0$ elsewhere, find?

(a). The total charge enclosed by the spherical surface r = 1mm.

(b). By using Gauss's law, calculate the value of D_r (electric flux density) on the surface r = 1mm.

Best Wishes

Q1 :-

(a) $|F|$

$$F_P = -10a_x + 20(3-1)a_y = -10a_x + 40a_y$$

$$|F| = \sqrt{(-10)^2 + (40)^2} = 41.23$$

(b) $|G|$

$$G_P = (2(2)^2 + 3)a_x - 4a_y + (-4)a_z$$

$$= 24a_x - 4a_y - 4a_z$$

$$|G| = \sqrt{(24)^2 + (-4)^2 + (-4)^2} = 24.66$$

(c)

$$F - G = (-10 - 24)a_x + (40 - (-4))a_y + (0 - (...$$

$$= -34a_x + 44a_y + 4a_z$$

$$|F - G| = \sqrt{(-34)^2 + (44)^2 - (4)^2} = 55.75$$

$$a_{F-G} = \frac{-34a_x + 44a_y + 4a_z}{55.75}$$

$$= -0.61a_x + 0.79a_y + 0.072a_z$$

(d)

$$F + G = (-10 + 24)a_x + (40 + (-4))a_y + (0 + -(...$$

$$= -14a_x + 36a_y - 4a_z$$

$$|F + G| = \sqrt{(14)^2 + (36)^2 + (-4)^2} = 38.83$$

$$a_{F+G} = \frac{14a_x + 36a_y - 4a_z}{38.83} = 0.36a_x + 0.93a_y$$

(a)

$R_{AB} = r_B - r_A$

$x = r\sin\theta\cos\phi \quad y = r\sin\theta\sin\phi \quad z = r\cos$

to $A(x = 5\sin 110^\circ\cos 200^\circ, y = 5\sin 110^\circ\sin 200^\circ, z = 5\cos$

$B(x = 7\sin 30^\circ\cos 70^\circ, y = 7\sin 30^\circ\sin 70^\circ, z = 7\cos 3$

finaly $A(-4.42, -1.61, -1.71)$ to $B(1.20, 3.29, 6.06)$

Thus $B - A = 5.62 a_x + 4.90 a_y + 7.77 a_z$

(b)

$R_{PQ} = r_Q - r_P. = -5 a_x + 5 a_y + a_z$

$r = \sqrt{x^2+y^2+z^2}$

Then at P, $r = \sqrt{4+9+16} = 5.39$

$\theta = \cos^{-1}\left(\frac{4}{\sqrt{29}}\right) = 42.0^\circ$ $\quad \theta = \cos^{-1}\frac{z}{\sqrt{x^2+}}$

$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-3}{2}\right) = -56.3^\circ$

$R_{PQ}\cdot a_r = -5\sin(42^\circ)\cos(-56.3^\circ) + 5\sin(42^\circ)\sin(-56$

$+ 1\cos(42^\circ) = -3.90$

$R_{PQ}\cdot a_\theta = -5\cos(42^\circ)\cos(-56.3^\circ) + 5\cos(42^\circ)\sin$

$-1\sin(42^\circ) = -5.82$

$R_{PQ}\cdot a_\phi = -(-5)\sin(-56.3^\circ) - 5\cos(-56.3^\circ) = -1$

So finally

$R_{PQ} = -3.90 a_r - 5.82 a_\theta - 1.39 a_\phi$

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First convert a_ρ to cartesian coordinates at the point. Use $a_\rho = (a_\rho \cdot a_x)\, a_x + (a_\rho \cdot a_y)\, a_y$.

At M(1, 2, 3), $\rho = \sqrt{5}$, $\phi = \tan^{-1}(2) = 63.4°$

$r = \sqrt{14}$ and $\theta = \cos^{-1}(3/\sqrt{14}) = 36.7°$

So $a_\rho = \cos(63.4°)\, a_x + \sin(63.4°)\, a_y = 0.45 a_x + 0.8$

Then $D \cdot a_\rho$

$(5 a_r - 3 a_\theta + 4 a_\phi) \cdot (0.45 a_x + 0.89 a_y)$

$= 5(0.45) \sin\theta \cos\phi + 5(0.89) \sin\theta \sin\phi - 3(0.45) \cos\theta \cos$

$- 3(0.89) \cos\theta \sin\phi + 4(0.45)(-\sin\phi) + 4(0.89) \cos\phi$

$= 0.59$

Q3 :-

Arrange the charges in the xy plane at Location (4, 4), (4, -4), (-4, 4) and (-4, -4). Then the fifth charge will be on the z axis at Location z = 4 which puts it at 8 cm distance from the other. By symmetry, the force on the fifth charge will be z-directed, and will be four times the z component of force produced by each of the four other charges.

$$F = \frac{4}{\sqrt{2}} \times \frac{q^2}{4\pi \epsilon_0 d^2} = \frac{4}{\sqrt{2}} \times \frac{(10^{-8})^2}{4\pi (8.85 \times 10^{-12})(0.0}$$

$= 4.0 \times 10^{-4}$ N

(a)

$$Q = \int_0^{2\pi}\int_0^{\pi}\int_0^{0.001} 2e^{-1000r} r^2 \sin\theta \, dr\, d\theta\, d\phi$$

integration over the angles gives a factor of 4π

$$Q = 8\pi\left[-\frac{r^2 e^{-1000r}}{1000}\Big|_0^{0.001} + \frac{2}{1000}\frac{e^{-1000r}}{(1000)^2}(-1000r-1)\Big|_0^{0.001}\right]$$

$$= 4.0 \times 10^{-9}\ nC$$

(b)

$$4\pi r^2 D_r = Q$$

$$D_r = \frac{Q}{4\pi r^2} = \frac{4.0\times10^{-9}}{4\pi(0.001)^2} = 3.2\times10^{-4}\ nC/m^2$$

ar !

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بسم الله الرحمن الرحيم

جامعة السودان للعلوم والتكنولوجيا

كلية الهندسة – قسم الهندسة الالكترونية

المادة: الحقول الكهرومغناطيسية

اختبار رقم (1)

الفصل الدراسي السادس	ثالثة الكترونيات	أستاذ المادة: د.محمد حسين	الزمن: ساعة

Question 1:

Volume charge density is given by $\rho_v = (\rho^2 - 10^{-4})\, z \sin 2\phi \quad c/m^3$ Calculate the total charge in the region $0.05 \le \rho \le 0.02,\ 0 \le \phi \le \frac{1}{2\pi} \quad 0 \le z \le 0.04$

Question 2:

A charge Q_0, located at the origin in free space, produces a field for which $E_z = 1\ KV/m$ at point p(-2,1,-1). Find Q_0.

Question 3:

Point charges of 50 nC each are located at A(1,0,0), B(-1,0,0), C(0,1,0) and D(0,-1,0) in free space. Find the total force on the charge at A.

"مع التمنيات بالتوفيق"

Question 1.

$$Q = \int_{Vol} \rho \, dv \quad \text{Then} \quad Q = \int_{\rho=0.02}^{0.05} \int_{z=0}^{0.04} \int_{\phi=0}^{\frac{1}{2\pi}} (\rho^2 - 10^{-4}) z \sin \rho \, d\rho \, d\phi$$

$$Q = \int_{\rho=0}^{0.05} \int_{z=0}^{0.04} \int_{\phi=0}^{\frac{1}{2\pi}} (\rho^3 - 10^{-4}\rho) z \sin\phi \, d\rho \, d\phi \, dz$$

$$= \int_{\rho=0}^{0.05} (\rho^3 - 10^{-4}\rho) d\rho \int_{z=0}^{0.04} z \, dz \int_{\phi=0}^{\frac{1}{2\pi}} \sin 2\phi \, d\phi$$

$$= \left[\frac{1}{4}\rho^4 - \frac{1}{2} 10^{-4} \rho^2\right]_{\rho=0}^{0.05} * \left[\frac{z^2}{2}\right]_{z=0}^{0.04} * \left[-\frac{1}{2}\cos 2\phi\right]_{\phi=0}^{\frac{1}{2\pi}}$$

$= 16.88pc$

Question 2.

The field at p will be

$$E_p = \frac{Q_0}{4\pi \epsilon |R_p|^2} a_p \qquad |R_p| = \sqrt{(-2)^2 + (1)^2 + (-1)^2} = \sqrt{6}$$

$$a_p = \frac{R_p}{|R_p|} = \frac{-2a_x + a_y - a_z}{\sqrt{6}}$$

$$E_p = \frac{Q_0}{4\pi\epsilon_0 \times 6}\left[\frac{-2a_x + a_y - a_z}{\sqrt{6}}\right]$$

Since the z componet is value 1 Kv/m, we find

$$Q_0 = -4\pi \epsilon_0 \times 6^{\frac{3}{2}} \times 10^3 = -1.63 \, \mu C$$

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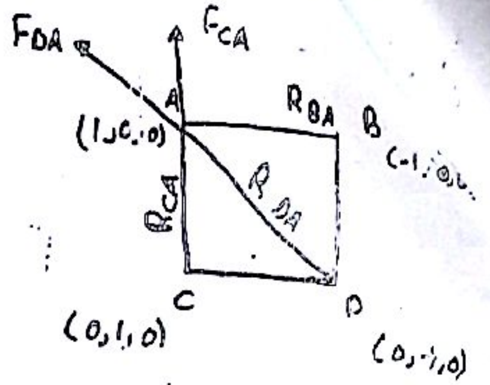

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 |R|^2} a_{12}$$

$R_{BA} = r_A - r_B = (1-(-1))a_x + (0-0)a_y + (0-0)a_z = 2a_x$

$|R_{BA}| = \sqrt{2^2+0^2+0^2} = \sqrt{4} = 2$, $a_{BA} = \frac{R_{BA}}{|R_{BA}|} = \frac{2a_x}{2} = a_x$

$R_{CA} = r_A - r_C = (1-0)a_x + (0-1)a_y + (0-0)a_z = a_x - a_y$

$|R_{CA}| = \sqrt{1^2+(-1)^2+0^2} = \sqrt{2}$, $a_{CA} = \frac{R_{CA}}{|R_{CA}|} = \frac{a_x - a_y}{\sqrt{2}} = \frac{a_x}{\sqrt{2}} - \frac{a_y}{\sqrt{2}}$

$R_{DA} = r_A - r_D = (1-0)a_x + (0-(-1))a_y + (0-0)a_z = a_x + a_y$

$|R_{DA}| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$, $a_{DA} = \frac{R_{DA}}{|R_{DA}|} = \frac{a_x + a_y}{\sqrt{2}} = \frac{a_x}{\sqrt{2}} + \frac{a_y}{\sqrt{2}}$

$$F_{BA} = \frac{Q_A Q_B}{4\pi\epsilon_0 |R_{BA}|^2} a_{BA}$$

$$F_{BA} = \frac{50 \times 50 \times 10^{-9}}{4\pi \times \frac{1}{36\pi}(4)} (a_x) = 5.68 a_x \mu N$$

$$F_{CA} = \frac{Q_C Q_A}{4\pi\epsilon_0 |R_{CA}|^2} a_{CA}$$

$$F_{CA} = \frac{50 \times 50 \times 10^{-9}}{4\pi \times \frac{1}{36\pi}(2)} \left(\frac{1}{\sqrt{2}} a_x - \frac{1}{\sqrt{2}} a_y\right) = 8.04 a_x \mu - 8.0$$

$$F_{DA} = \frac{Q_D Q_A}{4\pi\epsilon_0 |R_{DA}|^2} a_{DA}$$

$$F_{DA} = \frac{50 \times 50 \times 10^{-9}}{4\pi \times \frac{1}{36\pi}(2)} \left(\frac{1}{\sqrt{2}} a_x + \frac{1}{\sqrt{2}} a_y\right) = 8.04 a_x \mu + 8.0$$

$F_A = F_{BA} + F_{CA} + F_{DA}$

$= 21.8 a_x \mu N$

بسم الله الرحمن الرحيم

SUDAN UNIVERSITY OF SCIENCE AND TECHNOLOGY

College of Engineering – School of Electronics

Subject: Electromagnetic Field

Test no. (1)

Time: One hour	Lecturer: Dr. Mohamed Hussien	Semester: 6.	Class: 3rd year

Answer Only Two questions

Question 1:

It is known that the potential is given as $V = 90z^{4/3}$ volt at the plane ($z = 0$). Assume free space conditions. Find the electrical field intensity (E) the electrical flux density (D) and the volume charge density (ρ_v) as functions on z.

Question 2:

In cylindrical coordinates with $(\rho, \phi) = E_\rho(\rho, \phi)a_\rho + E_\phi(\rho, \phi)a_\phi$, the differential equation describing the direction of streamline is $\frac{E_\rho}{E_\phi} = d\rho/(\rho d\phi)$ in any z constant plane. Derive the equation of the streamline passing through point $p(2, 30^0, 0)$ in the field $E = \rho\cos(2\emptyset)_{a_\rho} - \rho\sin(2\emptyset)_{a_\phi} v/m$.

Question 3:

Evaluate both side of Stoke's theorem for the field $H = (10\sin\theta)a_\phi$ A/m and the rectangular path around the region $0 \le \theta \le 90^0$, $0 \le \phi \le 90^0$, $r = 3$.

All the Best

Dr. Mohamed Hussien

Question 1:

a) $E = -\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$

$= -120\, z^{1/3}\, a_z$ V/m $= -120\, z\, a_z$

b) $D = \epsilon_0 E = 1.062\, nC/m^3$

$a_x + z x^2 a_y$

*

c) $\rho_v = \nabla \cdot D = -280\, z^{10/3}\, nC/m^3$

Question 2:

$$\frac{E_\rho}{E_\phi} = \frac{d\rho}{(\rho d\phi)} \quad \text{and } E_\rho = \rho \cos(2\phi) \ldots \phi \ldots E_\phi = -\rho \sin(2\phi)$$

$$\frac{d\rho}{(\rho d\phi)} = \frac{\rho \cos 2\phi}{-\rho \sin 2\phi} \Rightarrow 2\frac{d\rho}{\rho} = \left(-2\frac{\cos 2\phi}{\sin 2\phi}\right) d\phi$$

$$\Rightarrow 2\ln(\rho) = -\ln(\sin 2\phi) + \ln(c)$$

$$\therefore \ln(\rho^2) + \ln(\sin 2\phi) = \ln(c)$$

$$\Rightarrow \ln(\rho^2 \sin 2\phi) = \ln(c) \Rightarrow \rho^2 \sin 2\phi = c$$

at $(2, 30^\circ, 0) \Rightarrow c = (2)^2 \sin 2(30) = 2\sqrt{3} \Rightarrow$

$$\therefore \rho^2 \sin 2\phi = 2\sqrt{3}.$$

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بسم الله الرحمن الرحيم

SUDAN UNIVERSITY OF SCIENCE AND TECHNOLOGY

College of Engineering – School of Electronics

Subject: Electromagnetic Field | Test no. (1)

Time: One hour | Lecturer: Dr. Mohammed Hussien | Semester: 6 | Class: 3rd year

Answer All questions

Question 1:

Point charges of 50nC each are located at A(1, 0, 0), B(−1, 0, 0), C(0, 1, 0) and D(0, −1, 0) in free space. Find the total force on the charge at A.

Question 2:

Given the electric field $E = (4x - 2y)_{a_x} - (2x + 4y)_{a_y}\ v/m$, find the equation of that streamline passing through point (2,3, −4), then find a unit vector a_E specifying the direction of E at (3, −2,5).

Question 3:

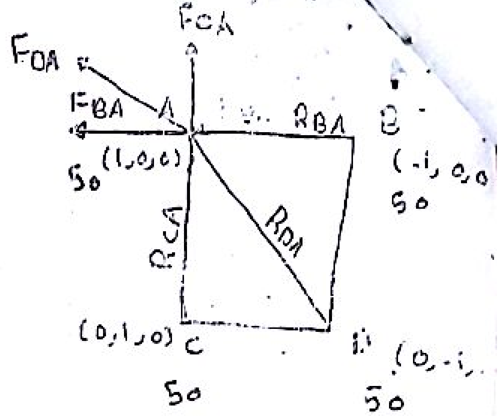
A non-uniform volume charge density, $\rho_v = 120r$ C/m³, lies within the spherical surface in spherical coordinate system. Find (a) the electric flux density D_r everywhere. (b) the electric flux density D_r at r = 1m.

All the Best

Dr. Mohamed Hussien

Solution

Question 15



$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 |R|^2} a_{12}$$

$R_{BA} = r_A - r_B = (1-(-1))a_x + (0-0)a_y + (0-0)a_z = 2a_x$

$|R_{BA}| = \sqrt{2^2+0^2+0^2} = \sqrt{4} = 2$, $a_{BA} = \frac{R_{BA}}{|R_{BA}|} = \frac{2a_x}{2} = a_x$

$R_{CA} = r_A - r_C = (1-0)a_x + (0-1)a_y + (0-0)a_z = a_x - a_y$

$|R_{CA}| = \sqrt{1^2+(-1)^2+0^2} = \sqrt{2}$, $a_{CA} = \frac{R_{CA}}{|R_{CA}|} = \frac{a_x - a_y}{\sqrt{2}} = \frac{a_x}{\sqrt{2}} - \frac{a_y}{\sqrt{2}}$

$R_{DA} = r_A - r_D = (1-1)a_x + (0-(-1))a_y + (0-0)a_z = a_x + a$

$|R_{DA}| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$, $a_{DA} = \frac{R_{DA}}{|R_{DA}|} = \frac{a_x + a_y}{\sqrt{2}} = \frac{a_x}{\sqrt{2}} +$

$$F_{BA} = \frac{Q_A Q_B}{4\pi\epsilon_0 |R_{BA}|^2} a_{BA}$$

$$F_{BA} = \frac{50\times10^{-9}\times 50\times10^{-9}}{4\pi\times\frac{1}{36\pi}\times10^{-9}(4)}(a_x) = 5.68 a_x \mu N$$

$$F_{CA} = \frac{Q_C Q_A}{4\pi\epsilon_0 |R_{CA}|^2} a_{CA}$$

$$F_{CA} = \frac{50\times10^{-9}\times 50\times10^{-9}}{4\pi\times\frac{1}{36\pi}\times10^{-9}(2)}\left(\frac{1}{\sqrt{2}}a_x - \frac{1}{\sqrt{2}}a_y\right) = 8.04\mu a_x - 8.0$$

$$F_{DA} = \frac{Q_D Q_A}{4\pi\epsilon_0 |R_{DA}|^2} a_{DA}$$

$$F_{DA} = \frac{50\times10^{-9}\times 50\times10^{-9}}{4\pi\times\frac{1}{36\pi}\times10^{-9}(2)}\left(\frac{1}{\sqrt{2}}a_x + \frac{1}{\sqrt{2}}a_y\right) = 8.04\mu a_x + 8.04$$

$F_A = F_{BA} + F_{CA} + F_{DA}$

$= 21.8\, a_x \mu N$

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The equation of the streamline that passes through the p. (2, 3, -4), we write

$$\frac{dy}{dx} = \frac{E_y}{E_x} = -\frac{(2x+4y)}{(4x-2y)}$$

Thus

or $$2(x\,dy + y\,dx) = y\,dy - x\,dx$$

$$d(xy) = \frac{1}{2}d(y^2) - \frac{1}{2}d(x^2)$$

So $$\frac{1}{2}C_1^2 + 2xy = \frac{1}{2}y^2 - \frac{1}{2}x^2$$

or $$y^2 - x^2 = 4xy + C_2^2$$

Evaluating at (2, 3, -4) obtain:

$$9 - 4 = 24 + C_2$$

or $$C_2 = -19$$

$$y^2 - x^2 = 4xy - 19$$

b) a unit vector specifying the direction of E at (3, -2, 5)

$$E = [4(3) + 2(2)]a_x - [2(3) - 4(2)]a_y$$

$= 16a_x + 2a_y$. Then $|E| = \sqrt{16^2 + 4} = 16.12$ So

$$a_E = \frac{E}{|E|} = \frac{16a_x + 2a_y}{16.12} = 0.99a_x + 0.12a_y$$

a.) $\pi \int 20\, r^2 (r)\, dr$

$= 120\pi r^4$

D.

r

$Q = \int \rho_v \, dv$ $a \hat{r}$

$= \int 4\pi r^2 \, dr \, \rho_v$

$= 4\pi \int_0^r 120 r \cdot r^2 \, dr$

$= 120\pi r^4$

$D_r = \dfrac{120\pi r^4}{4\pi r^2} = 30 r^2$

b) $D_r = 30 (1) = 30 \ C/m^2$.